FLINS-ISKE
2024

Book of Abstracts

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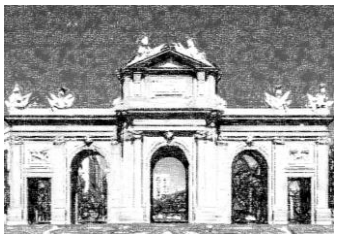
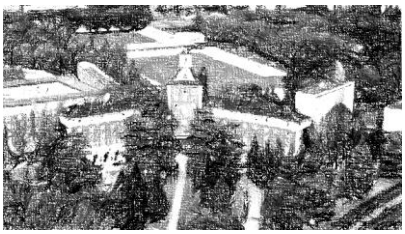

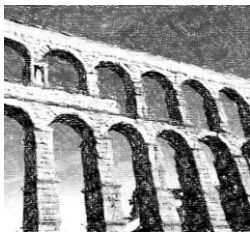

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Preface

Researchers were invited to submit an extended abstract of up to 2 pages. This extended abstract included the title, authors, affiliation, standard EasyChair abstract, keywords, body of the extended abstract, and key references.

The topics of the abstracts include artificial neural networks, classification, clustering, computing with words, decision-making, evolutionary/genetic algorithms, expert systems, feature extraction, fuzzy systems, geographic information systems, intelligent systems, machine learning, optimization, soft computing, and applications related to these topics such as water, energy, and environmental management, etc.

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UNIVERSIDAD COMPLUTENSE MADRID

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Instituto de Matemática Interdisciplinar

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Inclusion (or subsethood) in type-2 fuzzy sets

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The aim of this work is the study of inclusion measures in type-2 fuzzy sets.

Keywords: Fuzziness; Type-2 fuzzy set; Inclusion.

1. Summary

According to Mizumoto[1] a type-2 fuzzy set (T2FS) A is characterized by a *membership function* $\mu_A\colon X \to \mathbf{M} = Map([0,1],[0,1])$, that is, $\mu_A(x)$ is a type-1 fuzzy set in the interval $[0,1]$ and also the membership degree of the element $x \in X$ in the set A. Therefore, $\mu_A(x) = f_x$ where $f_x\colon [0,1] \to [0,1]$. We denote by $\text{T2FS}(X) = Map(X, \mathbf{M})$ the set of all T2FS on a finite Universe X.

Moreover, although some authors consider the membership degree $\mu_A(x)$ of an element x in T2FS as any fuzzy set[2], it is also possible to perceive it as a label of the variable *truth*[3]. We strongly prefer the latter interpretation since we consider it better aligns with the meaning of T2FS. These labels are usually normal and convex, that is, $\sup\{f(x) : x \in [0,1]\} = 1$ and for any $x \leq y \leq z$ it holds $f(y) \geq f(x) \wedge f(z)$, respectively. In this work we only consider this type of labels.

Given these assumptions, we have that $A \subseteq^* B$ if and only if $\forall x \in X$ it holds $\mu_B^L(x) \leq \mu_A^L(x)$ and $\mu_A^R(x) \leq \mu_B^R(x)$, except in a set of measure zero, where for any $f \in [0,1]^{[0,1]}$ we have $f^L, f^R \in [0,1]^{[0,1]}$ defined as follows:

$$f^L(x) = sup\{f(y) : y \leq x\} \quad \text{and} \quad f^R(x) = sup\{f(y) : y \geq x\}.$$

Inspired by Kosko's work[4], we analyze the conditions needed to define an inclusion measure between type-1 fuzzy sets. These properties will be required by any inclusion measure between fuzzy sets or any of its extensions

(interval-valued or type-2 fuzzy sets).

Definition 1.1. Let A and B be T2FS with membership degrees $\mu_A(x)$ and $\mu_B(x)$, respectively, being normal and convex functions for each $x \in X$.

We say that a function $\mathcal{I}\colon T2FS(X) \times T2FS(X) \to [0,1]$ is an inclusion (or subsethood) measure in T2FS(X) if it satisfies the following axioms:

($\mathcal{I}1$) $\mathcal{I}(A, \mathcal{O}) = 0$ if $A \neq \mathcal{O}$, being $\mathcal{O}$ the empty T2FS, that is, $\mathcal{O}(x) = \bar{0}$ for all $x \in X$, where $\bar{0}(0) = 1$ and $\bar{0}(y) = 0\ \forall y \neq 0$.
($\mathcal{I}2$) $\mathcal{I}(A, B) = 1 \iff A \subseteq^* B$.
($\mathcal{I}3$) If $B \subseteq^* C \Longrightarrow \mathcal{I}(A, B) \leq \mathcal{I}(A, C)$.
($\mathcal{I}4$) If $A \subseteq^* B \subseteq^* C \Longrightarrow \mathcal{I}(C, A) \leq \mathcal{I}(B, A)$ and $\mathcal{I}(C, A) \leq \mathcal{I}(C, B)$.
($\mathcal{I}5$) Let $A^c \subseteq^* A$. $\mathcal{I}(A, A^c) = 0 \iff A = X$, being X the universal T2FS, that is, $X(x) = \bar{1}$ for all $x \in X$, where $\bar{1}(1) = 1$ and $\bar{1}(y) = 0\ \forall y \neq 1$.

The aim then is to find different inclusion measures in T2FS(X). Among others, we have $\mathcal{I}_1(A, B) = 1$ if $A = \mathcal{O}$, elsewhere $\mathcal{I}_1(A, B)$ equals to

$$\frac{\sum_{i=1}^{N} \left(1 - \int_0^1 \max(\mu_A(x_i)^L(y), \mu_B(x_i)^L(y))\, dy + \int_0^1 \min(\mu_A(x_i)^R(y), \mu_B(x_i)^R(y))\, dy\right)}{\sum_{i=1}^{N} \left(1 - \int_0^1 \mu_A(x_i)^L(y)\, dy + \int_0^1 \mu_A(x_i)^R(y)\, dy\right)}$$

Acknowledgements

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